

Current Phase Relation and transport properties of a Carbon Nanotube Coupled to Superconducting Leads

J. Basset,¹ R. Delagrangé,¹ R. Weil,¹ A. Kasumov,¹ H. Bouchiat,¹ and R. Deblock¹

¹*Laboratoire de Physique des Solides, Université Paris-Sud,
CNRS, UMR 8502, F-91405 Orsay Cedex, France.*

We propose a scheme to measure on the same sample the current phase relation and the differential conductance dI/dV of a superconducting junction in the normal and the superconducting states. This is done using a dc Superconducting Quantum Interference Device (dc SQUID) with two Josephson junctions in parallel with the device under investigation and three contacts. The measurements of the current phase relation and dI/dV are demonstrated experimentally on a small Josephson junction and a carbon nanotube junction. In this latter case, in a regime where the nanotube is well conducting, the measured non-sinusoidal current phase relation is consistent with the theory for a weak link whose transmission is extracted from the differential conductance in the normal state. This method is very promising for future investigations of the current-phase relation of more exotics SNS junctions.

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I. INTRODUCTION

The prediction that a dissipationless current should flow between two superconductors separated by a thin insulator barrier was made in 1962 by Josephson [1]. Since then many works extended the validity of this prediction to other kinds of weak links such as narrow constrictions of the superconductor thin film or superconductor - normal metal - Superconductor junctions (SNS)[2–6]. The supercurrent is expected to vary periodically with the phase difference of the two superconducting electrodes, with the maximum of this supercurrent being called the critical current of the junction. Probing the current phase relation (CPR) of the junction implies to phase bias it [5, 6, 9]. This is often done by including the weak link into a loop through which a magnetic flux is applied, thus constituting a SQUID. In the ac SQUID configuration only one junction is present in the loop whereas in the dc SQUID two junctions are present. In such configurations, the weak link is either short circuited (ac SQUID) or placed in parallel with another junction (dc SQUID) so that no direct conductance measurement can be made.

In this work we propose a scheme to measure on the same sample the current phase relation and the differential conductance. When the system is in the normal state, one wants to be able to measure the differential conductance of the weak link. This is needed to properly characterize the weak link. It has also a practical importance for carbon nanotube based weak links : it helps selecting the interesting devices *i.e.* with relatively high conductance at room temperature. In the superconducting state, the weak link should be phase biased in order to measure the current-phase relation. We want also to be able to measure the differential conductance $dI/dV(V)$ of the weak link and thus voltage biased it. To do so our proposal is to modify the dc SQUID geometry by introducing two reference Josephson junctions with a

contact in between in one branch of the SQUID and the weak link under investigation in the other one.

The paper is organized as follows. In section II we will consider the influence of this geometry on the measurement of the CPR. We will then present experimental results obtained when the weak-link is a small Josephson junction (section III) or a well conducting carbon nanotube (section IV).

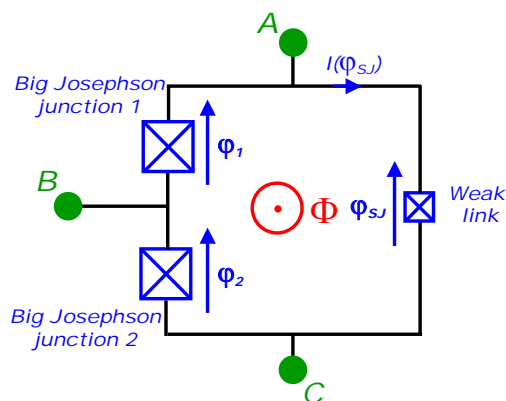


FIG. 1: Schematic picture of the dc SQUID with three contacts. Each junction has a superconducting phase difference φ_1 , φ_2 and φ_{SJ} . Contacts *A* and *C* are respectively the SQUID source and drain. Contact *B* is an additional central contact between the two almost identical big junctions.

II. CURRENT PHASE RELATION MEASUREMENT WITH A SQUID GEOMETRY

The aim of this work is to determine the CPR of a weak link, for *e.g.* a carbon nanotube junction, with the

possibility to also measure its differential conductance dI/dV in the normal and superconducting states. In order to measure the CPR of the weak-link one needs to control its phase difference *i.e.* realize a phase bias. This phase bias is also needed to probe the phase dependent Andreev bound state of the weak link [7, 8]. The phase bias for measuring the CPR of the weak link can be done by inserting it in a superconducting loop, in a dc SQUID geometry. The weak link is then in parallel with a Josephson junction. In this geometry to unambiguously measure the conductance of one branch independently of the other, one technique consists in measuring the conductance of the SQUID, then opening one branch of the SQUID and measuring the conductance of the remaining branch and by subtraction, extract the conductance of the opened arm. This technique is commonly used for break junctions [9] and can be adapted to a gate tunable system with a strong reduction of the conductance [10–13]. A second technique is to use the hysteretic IV characteristics of the Josephson junction to obtain information on the small weak link. This however does not allow to extract the low voltage bias behaviour due to the finite bias voltage at which the phase is retrapped.

To overcome these difficulties, we consider a SQUID geometry involving three contacts (Figure 1). One branch contains the weak link (small Josephson junction, carbon nanotube quantum dots,...) under investigation. The other supports two almost identical large Josephson junctions in series with a central contact (depicted by the letter *B*). The interest of this device is three-fold. First, the SQUID geometry allows to extract the current phase relation of the weak link from the modulation with magnetic flux of the switching current. Second, the third contact between the two large junctions allows to measure the normal state resistance of the weak link at room temperature. This is particularly useful for selecting hybrid junctions with low enough resistance and more generally to check the quality of the contact made on a sample. Third, one can use the same third contact to measure the differential conductance of the weak link in the superconducting state without relying on the hysteresis loop of the junctions in parallel with the weak link.

In the following we will consider the difference between the three junctions SQUIDs and the two junctions SQUID.

A. Resistively Capacitively Shunted Junction (RCSJ) model

The current at which a Josephson junction switches from the zero resistance state to the resistive state (switching current I_S) is always smaller than the critical current I_C . This is related to the dynamics of the superconducting phase across the junction and has been widely studied in the last decades [2, 3]. In particular it was shown that the switching current depends on the

electromagnetic environment in which the junction is embedded and can be described within the *RCSJ* model. The junction is then considered as a perfect Josephson element in parallel with a *RC* circuit. The superconducting phase difference across the junction is noted φ .

When the junction is current biased with a current $I = sI_C$, with I_C the critical current of the junction (determined by theory [14]) and $s \in [-1, 1]$ a real number, one has :

$$I = I_C \sin(\varphi) + \frac{V}{R} + C \frac{dV}{dt}. \quad (1)$$

Using the Josephson relation $2eV = \hbar d\varphi/dt$ one gets [2] :

$$\phi_0 C \frac{d^2\varphi}{dt^2} + \frac{\phi_0}{R} \frac{d\varphi}{dt} + I_C \sin(\varphi) = I \quad (2)$$

with $\phi_0 = \hbar/2e$. It is analog to the equation of motion of a particle of mass $m = \phi_0^2 C$ moving along the φ axis in the effective washboard potential $U(\varphi)$:

$$U(s, \varphi) = -\phi_0 I_C \cos(\varphi) - \phi_0 I \varphi, \quad (3)$$

with a dissipation related to the quality factor $Q = \omega_P RC$ of the system, with $\omega_P = \sqrt{2eI_C/\hbar C}$ the plasma frequency of the junction. When the applied current is below the critical current, *i.e.* $s < 1$, the fictitious particle is trapped into a local minimum of the potential where it oscillates at $\omega_P(s)$. These oscillations are damped with a time scale Q/ω_P . However, due to fluctuations in the current $\delta I(t)$, the particle can escape this local minimum leading to a finite dc voltage drop across the junction. Depending on the nature of escape process, thermal or quantum, the escape rate has different expressions.

a. Thermal escape The probability for the fictitious particle to escape the well at a given current is given by $P_t(s) = 1 - e^{-\Gamma(s)t}$ with :

$$\Gamma(s) = a(Q) \frac{\omega_P(s)}{2\pi} e^{-\Delta U(s)/k_B T_{esc}}. \quad (4)$$

$\omega_P(s)$ is the plasma frequency for current sI_C , the function $a(Q)$ takes into account the friction, T_{esc} corresponds to the escape temperature (temperature of the electromagnetic environment) and $\Delta U(s)$ the height of the potential barrier at the current $I = sI_C$. The friction determines the value of $a(Q)$ and the regime of the junction (Table I) [15–17].

Damping	Validity range	$a(Q)$
Underdamped, low	$Q > 1, \frac{2\pi\Delta U}{k_B T} \frac{\omega_0}{Q\omega_P} < 1$	$2\pi \frac{\Delta U}{k_B T} \frac{\omega_0}{Q\omega_P} < 1$
Underdamped, mod	$Q > 1, \frac{2\pi\Delta U}{k_B T} \frac{\omega_0}{Q\omega_P} > 1$	1
Overdamped	$Q < 1$	$\frac{Q\omega_P}{\omega_0}$

TABLE I: Criterion for crossover between different damping regimes and the prefactor $a(Q)$ of the tunneling rate formula 4. In these formulas $\omega_0 = \omega_P(s = 0)$

b. Quantum escape In addition to thermal escape one has to consider quantum escape. Indeed since the phase is a quantum variable it may escape the potential well via quantum tunneling. The tunneling rate is well approximated in the underdamped regime by [18, 19] :

$$\Gamma_{Tunnel}(s) = 6^{3/2} \sqrt{\pi} \omega_P(s) \sqrt{\frac{\Delta U(s)}{\hbar \omega_P(s)}} e^{\frac{-36 \Delta U(s)}{5 \hbar \omega_P(s)}}. \quad (5)$$

The crossover temperature between thermal and quantum escape is given by $T_{cross} = \hbar \omega_P(s=0)/2\pi k_B$.

In the following we will concentrate on the thermally activated behaviour, which is relevant for our experiments.

B. Weak link embedded in a SQUID

When the junction investigated is in parallel with a Josephson junction the previous model is modified. The capacitance C and the conductance $1/R$ are now respectively the sum of the capacitance and the conductance of the Josephson junction and the probed junction. In this geometry, due to the fluxoid quantization, the phase difference of the small junction φ_{SJ} is related to the phase difference φ across the Josephson junction by :

$$\varphi_{SJ} = \varphi - 2\pi\phi/\phi_0 + 2n\pi \quad (6)$$

where ϕ is the magnetic flux through the SQUID loop and n an integer. We will note $I_{SJ}f(\varphi_{SJ})$ the current phase relation of the junction, with I_{SJ} its critical current. In this case the equation for the phase dynamics is :

$$\phi_0 C \frac{d^2\varphi}{dt^2} + \frac{\phi_0}{R} \frac{d\varphi}{dt} + I_C \sin(\varphi) + I_{SJ}f(\varphi - 2\pi\phi/\phi_0) = I \quad (7)$$

Similarly to the previous section, this corresponds to the dynamics of a fictitious particle evolving in the potential $U(\varphi, I)$, which is now given by :

$$U(\varphi, I)/\phi_0 = -I_C \cos(\varphi) + I_{SJ}F(\varphi - 2\pi\phi/\phi_0) - I\varphi, \quad (8)$$

with $F(\varphi)$ a primitive function of the current phase function f . This potential is modified by the magnetic flux ϕ applied to the SQUID loop. This is what allows to extract the current phase relation from the modulation of the switching current of the SQUID. We will now consider the case of a SQUID with two Josephson junctions and a weak link.

C. Asymmetric SQUID with a central contact

In the SQUID geometry shown in fig. 2, we denote φ_1 , φ_2 the phase difference across the two reference junctions and φ_{SJ} the phase across the weak link. Due to fluxoid quantization, one has now :

$$\varphi_{SJ} - (\varphi_1 + \varphi_2) = -2\pi\phi/\phi_0 + 2n\pi \quad (9)$$

The critical current of the two big junctions are noted I_{C1} and I_{C2} .

In the following, we will consider the three junctions in the framework of the RCSJ model, as shown in figure 2. One obtains the following equations relating the currents

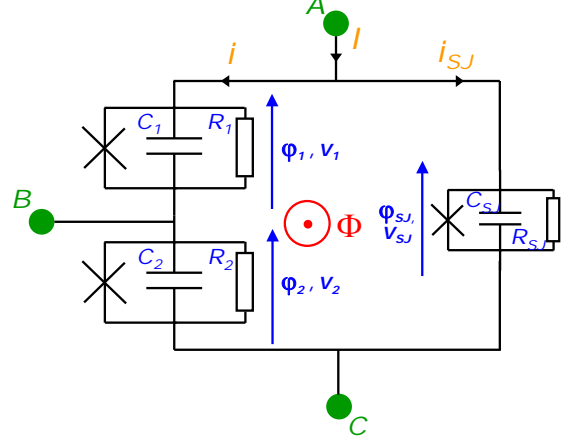


FIG. 2: RCSJ model for the asymmetric SQUID considered in the experiment.

and the phase difference for each junction :

$$i = I_{C1} \sin(\varphi_1) + \frac{\phi_0}{R_1} \frac{d\varphi_1}{dt} + \phi_0 C_1 \frac{d^2\varphi_1}{dt^2} \quad (10)$$

where the equation for junction 2 and the weak link can be obtained by replacing "1" by "2" or "SJ". In the following we will neglect the dynamics of the phase across the small junction compared to the one of the big junctions. This corresponds to neglect the small capacitance and conductance of the weak link compared to the ones of the Josephson junction. In this case the previous equations can be recast as :

$$\begin{aligned} I &= I_{C1} \sin(\varphi_1) + I_{SJ}f(\varphi_{SJ}) + \frac{\phi_0}{R_1} \frac{d\varphi_1}{dt} + \phi_0 C_1 \frac{d^2\varphi_1}{dt^2} \\ I &= I_{C2} \sin(\varphi_2) + I_{SJ}f(\varphi_{SJ}) + \frac{\phi_0}{R_2} \frac{d\varphi_2}{dt} + \phi_0 C_2 \frac{d^2\varphi_2}{dt^2} \end{aligned} \quad (11)$$

These equations correspond to the dynamics of a fictitious particle evolving in the 2D potential :

$$\begin{aligned} U(\varphi_1, \varphi_2, I)/\phi_0 &= -I_{C1} \cos(\varphi_1) - I_{C2} \cos(\varphi_2) \\ &+ I_{SJ}F(\varphi_1 + \varphi_2 - 2\pi\phi/\phi_0) - (\varphi_1 + \varphi_2) \cdot I \end{aligned} \quad (12)$$

In this situation, depending on the nature of the current noise through the SQUID two regimes can be reached. The first one, called hereafter "2D dynamics" corresponds to variation of the phases of each big junction which is independent of one another. The other one, called hereafter "1D dynamics", corresponds to a strongly coupled variations of the two phases, which dynamics is synchronized. In this latter situation, since

$I_{C1} = I_{C2} = I_0$ and supposing that the two junctions are identical we will make the assumption that, at anytime before the switching, $\varphi_1 = \varphi_2 = \varphi$. Then one has to study the dynamics of the phase in the effective potential :

$$U(\varphi, I) = -I_0 \cos(\varphi) + \frac{1}{2} I_{SJ} F(2\varphi - 2\pi\phi/\phi_0) - I\varphi \quad (13)$$

This has to be contrasted with the case of the regular SQUID where the potential is given by Eq. 8.

Due to the presence of the third contact, taking into account the current fluctuations in the dynamics of the SQUID is more complicated than in the dc SQUID geometry. This is done phenomenologically by incorporating an effective temperature T_{esc} which can be substantially different from the real electronic temperature of the experiment but also from the effective temperature in the standard dc SQUID geometry.

D. Comparison of the two SQUID geometries

The measurement of the current-phase relation of the weak link is deduced from the modulation of the switching current of the SQUID in the limit where the supercurrent of the Josephson junctions is much higher than the one of the supercurrent of the weak-link. We note the switching current I_S , its average value $\langle I_S \rangle$ and $\Delta I_S(\Phi)$ the magnetic field dependent part, so that : $I_S = \langle I_S \rangle + \Delta I_S(\Phi)$. In the very low temperature limit $\Delta I_S(\Phi)$ is related to the current phase relation of the weak link and $\langle I_S \rangle$ is the switching current of the big Josephson junctions. In the next part we address the relation between the measured modulation and the real current phase relation. To do so we have calculated the expected modulation of the moderately underdamped SQUID in the thermal escape regime as a function of temperature comparing the cases of a two junctions and three junctions SQUID. We suppose that the weak link in one branch of the dc SQUID has a sinusoidal current-phase relation, i.e. $f(\varphi_{SJ}) = \sin \varphi_{SJ}$ in eq. 7 and 11.

We consider a SQUID submitted to a current bias increasing linearly with time at a rate dI/dt . In this case the probability for the system to have switched to the resistive state at a current I is [15–17] :

$$W(I) = 1 - \exp \left(- \int_0^I dI' \frac{\Gamma(I')}{dI/dt} \right)$$

The switching current is determined by solving numerically $W(I) = 1/2$ which is equivalent for a moderately underdamped SQUID in the thermal escape regime to :

$$\int_0^s ds' \omega_P(s') \exp \left(- \frac{\Delta U(s')}{k_B T_{esc}} \right) = \frac{2\pi}{I_C} \frac{dI}{dt} \ln(2) \quad (14)$$

with $s = I/I_C$. To perform the calculation of the switching current we have taken the following parameters :

$I_0 = 300\text{nA}$, $C_{Tot} = 50\text{fF}$, $dI/dt = 500\mu\text{A/s}$ and compared the switching current and the amplitude of the modulation (Fig. 3) for the case with one junction and two junctions.

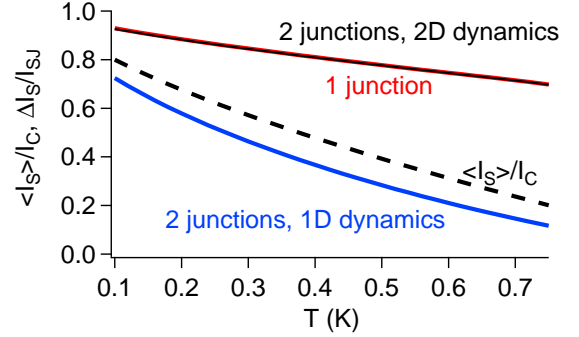


FIG. 3: Calculated $\langle I_S \rangle$ in unit of I_C (black dashed line) and amplitude of ΔI_S in unit of the supercurrent I_{SJ} of the weak link for a moderately underdamped SQUID in the thermal escape regime with the parameters $I_C = 300\text{nA}$, $C_{Tot} = 50\text{fF}$ and $dI/dt = 500\mu\text{A/s}$. We have chosen $I_{SJ} = 0.05I_C$ but the calculated ratio $\Delta I_S/I_{SJ}$ is independent of I_{SJ} for $I_{SJ} < 0.1I_C$. Different cases are shown : the case of a SQUID with a single big Josephson junction (red curve) or with two big Josephson junctions with 1D dynamics (black curve) or 2D dynamics (blue curve). Note that the black and red curve are nearly perfectly superimposed.

We see that the situation with a regular SQUID and a SQUID with two big junctions in the 2D dynamics regime are nearly identical. On the other hand the situation in the 1D limit is quite different and leads in particular to a smaller modulation of the SQUID supercurrent. This is related to the reduction by a factor 2 of the effect of the weak-link in the effective potential (equation 13) and the 2φ phase dependence of this term. We thus see that the dynamics of the phase in the SQUID geometry has important consequences regarding the amplitude of the modulation of the supercurrent.

III. CURRENT-PHASE RELATION AND CONDUCTANCE MEASUREMENT OF A JOSEPHSON JUNCTION

To test the validity of the theoretical analysis of section II and check the feasibility of the extraction of the current phase relation in a two junctions SQUID configuration we have measured two samples where the weak link consists of small Josephson junctions (Fig. 4). The junctions have been fabricated by electron beam lithography on the surface of oxidized wafers and shadow evaporation. For sample 1 the sequence of deposited materials is $\text{Pd}(4\text{nm})/\text{Al}(70\text{nm})/\text{AlO}_x/\text{Al}(120\text{nm})$ and is the same as the one used for samples including a carbon nanotube (see section IV). Taking benefit of the three terminals configuration, the resistance of the junctions can be

measured : $R_1 = 1.48k\Omega$, $R_2 = 1.46k\Omega$ and $R_{SJ} = 27k\Omega$ (Pd/Al superconducting gap $\Delta = 160\mu\text{eV}$, Al superconducting gap $\Delta = 240\mu\text{eV}$). This leads to calculated supercurrent values $I_{C1} = 232\text{nA}$, $I_{C2} = 235\text{nA}$ and $I_{SJ} = 12.7\text{nA}$. On the other hand sample 2 was fabricated using only aluminum ($\text{Al}(70\text{nm})/\text{AlO}_X/\text{Al}(120\text{nm})$) and has parameters : $R_1 = 2.33k\Omega$, $R_2 = 2.28k\Omega$ and $R_{SJ} = 18.0k\Omega$, superconducting gap $\Delta = 185\mu\text{eV}$, $I_{C1} = 125\text{nA}$, $I_{C2} = 127\text{nA}$ and $I_{SJ} = 16.1\text{nA}$.

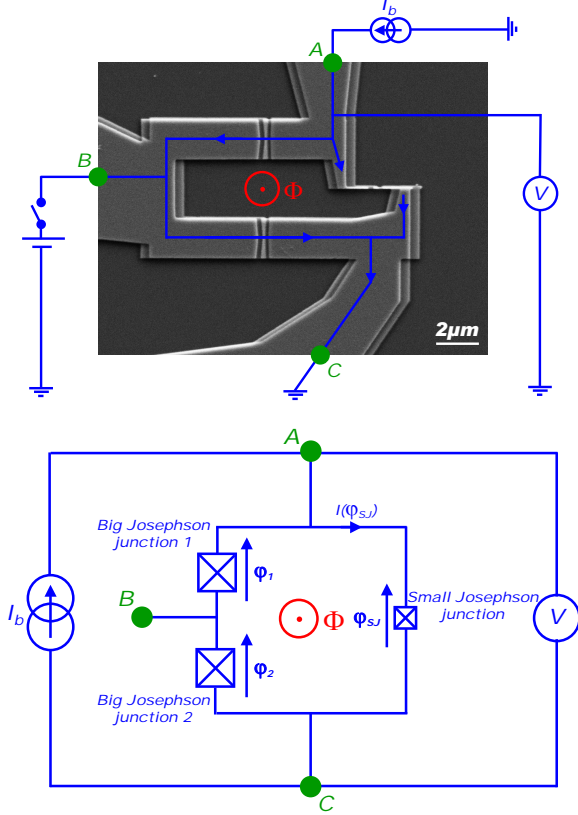


FIG. 4: (a) Scanning electron microscope picture of the asymmetric superconducting SQUID loop. (b) Equivalent circuit of the SQUID. The phase φ_1 , φ_2 and φ_{SJ} are linked to one another by the magnetic flux through the loop ϕ by relation $\varphi_{SJ} - (\varphi_1 + \varphi_2) = -2\pi\phi/\phi_0 + 2n\pi$ with $\phi_0 = h/2e$. Contacts A and C are respectively the SQUID source and drain. Contact B is the additional central contact between the two big junctions. It allows to determine the normal state resistance of each junctions.

A. Current Phase relation measurement

The switching current of the SQUID is measured by applying a current bias increasing linearly with time and recording the value of the current where the SQUID switches to a resistive state. This measurement is repeated several times in order to obtain an average value of the switching current. This procedure is repeated for

different values of magnetic field. This leads to the modulation with magnetic flux of the switching current of the SQUID (Fig. 5 for sample 1 and 2).

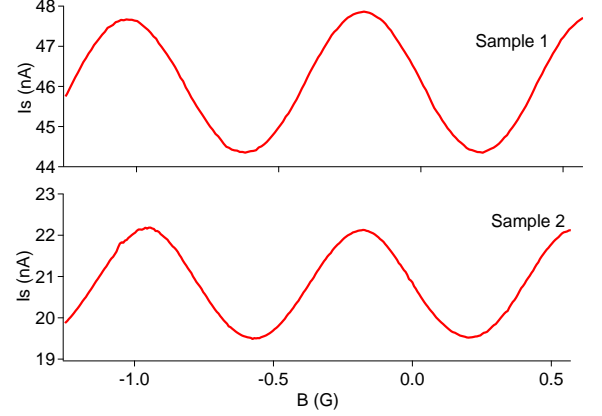


FIG. 5: Modulation of the switching current of the SQUID versus magnetic field for sample 1 ($T=230\text{mK}$) and 2 ($T=60\text{mK}$).

For sample 1 the experiment was done at a temperature $T = 230\text{mK}$. To explain the value of the switching current $I_S = 46\text{nA}$ compared to the value of the critical current $I_C = 234\text{nA}$ we have to take an effective temperature $T_{esc} = 460\text{mK}$ in relation 14. As noted before since it is more difficult to take into account the current noise in the two reference junctions squid geometry, this effective temperature can be substantially different from the case of the standard DC squid geometry. This unusually high value of phase temperature may also point out the fact that our experiment is not in the purely thermally activated regime but can also exhibit phase diffusion [20]. With this effective temperature we calculate the expected modulation of the switching current with a 1D or 2D dynamics to define which model is more relevant for our experiment (Fig. 6). The calculated modulation in the 2D case is nearly 5 times bigger than the measured one whereas the 1D case gives a good agreement with the experimental data. For sample 2, the 2D calculation overestimates by a factor 6 the measured switching current modulation. The 1D calculation is in agreement with the experiment within 30 % for the amplitude.

The sinusoidal shape of the current phase relation, expected for this Josephson junction, is measured correctly in the experiment. However the quantitative agreement with the presented theory is not satisfactory. The regime of 1D dynamics is closer to the experimental data. This result may motivate more involved calculations in this new three terminal SQUID structure.

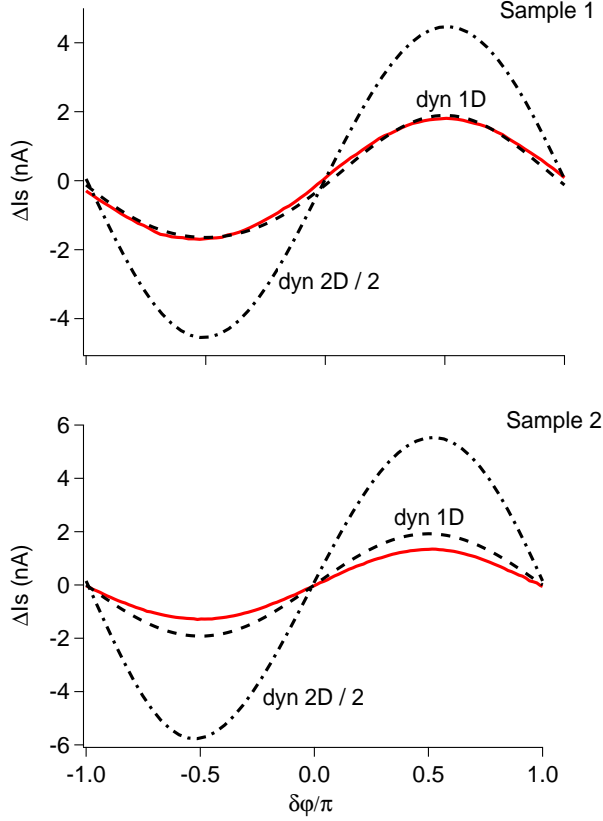


FIG. 6: Modulation of the switching current for sample 1 and 2 (plain red line) and comparison with calculated modulation for the two junction squid with 1D (black dashed line) or 2D dynamics (black dashed-dotted line). Note that the amplitude of the calculated curve in the 2D dynamics is divided by a factor 2 for better comparison.

B. dI/dV of the small junction in the superconducting state

To measure the differential conductance dI/dV in the superconducting state we apply a dc voltage bias on the SQUID (between A and C) and monitor the current flowing out in point C while maintaining the bias voltage at point B such that V_{BC} stay below the superconducting gap of the junction. By doing so it was possible to extract the differential conductance of the small Josephson junction with this scheme (Fig. 7) provided that the Josephson branch of the big Josephson junctions are suppressed by a magnetic flux equal to a flux quantum in the area of the big junctions.

In this last section we have demonstrated the possibility to measure on the same sample the current phase relation and the differential conductance in the superconducting state by using a SQUID geometry with two reference Josephson junctions and three contacts. Hereafter we use the same detection scheme to measure the current phase relation of a carbon nanotube quantum dot

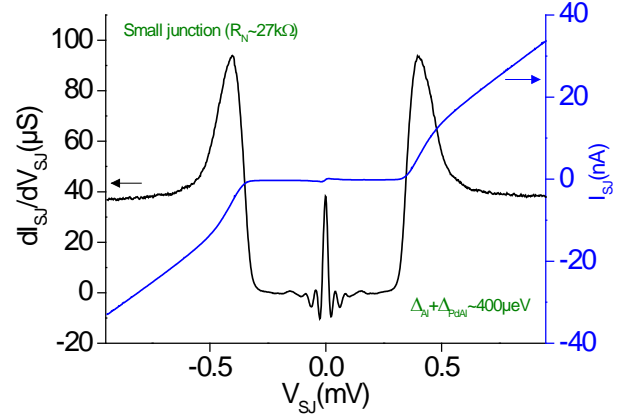


FIG. 7: Differential conductance dI_{SJ}/dV_{SJ} and dc characteristics $I_{SJ}(V_{SJ})$ of the small junction. The traces are obtained by fixing V_{BC} below the superconducting gap of the corresponding large junction. Superconducting gap is found to be $\Delta_{PdAl} + \Delta_{Al} = 400 \mu eV$ and the normal state resistance $R_{N,SJ} = 27 k\Omega$.

strongly coupled to superconducting leads.

IV. CURRENT PHASE RELATION OF A CARBON NANOTUBE QUANTUM DOT JUNCTION

A. Sample Fabrication

The design of the sample is similar to sample 1 described previously with the weak link constituted by a carbon nanotube junction. The carbon nanotube is grown by chemical vapor deposition [21] and is connected with $PdAl/AlO_X/Al$ contacts in the same run of deposition as the big junctions in parallel (Fig. 8). The three points measurements made at room temperature allows to determine the resistance of each junctions : $R_1 = 1.03 k\Omega$, $R_2 = 1.02 k\Omega$.

B. Normal state characterization of the carbon nanotube quantum dot

The normal state characterization of the carbon nanotube is achieved by first applying a magnetic field $B \approx 0.18 T$ which suppresses superconductivity in the contacts. The differential conductance dI/dV is then measured with a lock-in amplifier technique as a function of bias and back-gate voltages (Fig. 9). The nanotube is globally highly conducting with a maximum differential conductance approaching $4e^2/h$.

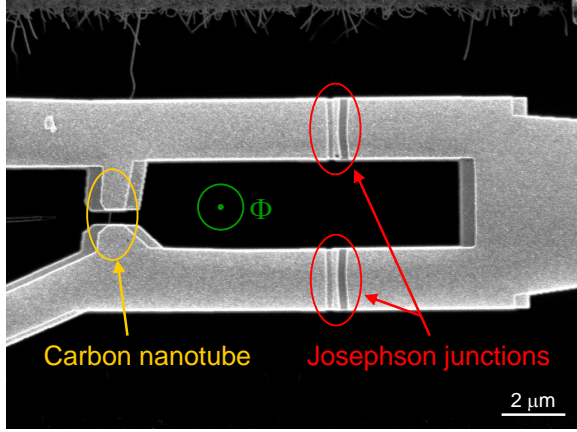


FIG. 8: Scanning electron microscope picture of the asymmetric SQUID used in the experiment. Junctions and carbon nanotube contacts are made of $PdAl/AlO_x/Al$ and are fabricated in the same step of metal deposition. Nanotube contacts are separated by $\approx 450nm$.

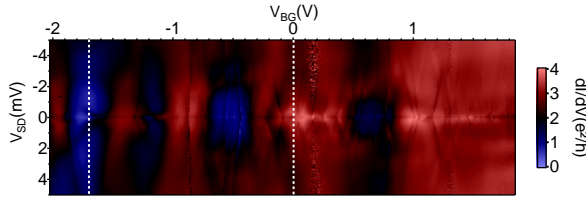


FIG. 9: Normal state stability diagram of the carbon nanotube quantum dot *i.e.* differential conductance $dIdV$ vs bias voltage V_{SD} and back-gate voltage V_{BG} .

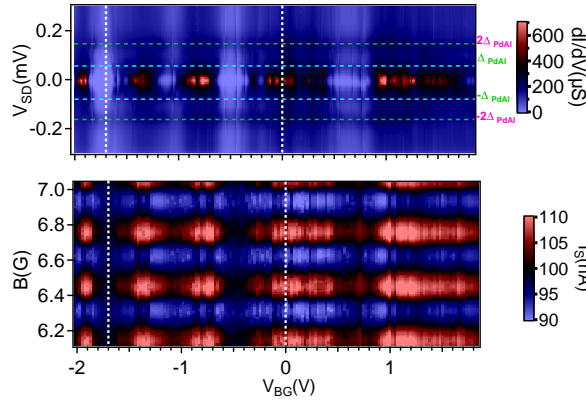


FIG. 10: Top panel : differential conductance dI/dV of the carbon nanotube quantum dot in the superconducting state. The two vertical dashed lines indicate cuts at a given backgate voltage shown in figure 11. The horizontal dashed lines indicate the position of the multiple Andreev reflections. Bottom panel : Modulation of the SQUID supercurrent *vs* the applied magnetic field B and the back-gate voltage.

C. Superconducting state characterization of the carbon nanotube quantum dot

To measure dI/dV in the superconducting state, we have reduced the magnetic field below the critical field of the contact and used the technique described in section III (Fig. 10). Two traces are also shown in fig 11a. We observe zero bias peaks in the regions of high normal state conductances and dips in the low conductance region. In addition to this, multiple Andreev reflections are also visible at fixed voltages $2\Delta/n$ with n an integer number ($n = \pm 1, \pm 2$ here). Finally, far away from the superconducting gap ($|V_{SD}| \gg 2\Delta$), the conductance remains constant.

D. Current phase relation measurement

In the superconducting state the SQUID exhibits a modulation of its supercurrent versus magnetic flux over the entire investigated range of gate voltage. This shows that the nanotube junction exhibits a supercurrent for those gate value. In order to perform a quantitative analysis we focus on two gate voltage values, one corresponding to a high value of the conductance in the normal state of the junction ($V_G = 0$ V) and one which is less conducting ($V_G = -1.7$ V). For each gate voltage we have measured both the current phase relation extracted from the modulation of the switching current (Fig. 11b) and the differential conductance dI/dV in the superconducting state (Fig. 11a). At certain gate voltages supercurrents as high as 12 nA were induced through the tube. Jointly, the current-phase relation exhibits an anharmonic behaviour.

We have tried to relate the shape and amplitude of the current-phase relation to the value of the conductance at zero bias in the normal state. We measured for $V_G = 0V$, $G = 3.19e^2/h$ and $G = 0.7e^2/h$ for $V_G = -1.7V$. Considering the carbon nanotube as a conductor with two spin degenerate conducting channels with the same transmission τ one gets $\tau = 0.79$ at $V_G = 0V$ and $\tau = 0.175$ at $V_G = -1.7V$. Such channels, with transmission τ_i between two superconducting contacts of gap Δ are expected to show a current-phase relation given by [22, 23] :

$$I(\varphi) = \sum_i \frac{e\tau_i\Delta}{2\hbar} \frac{\sin(\varphi)}{\sqrt{1 - \tau_i \sin^2(\varphi/2)}} \quad (15)$$

The experiment was done at a temperature $T = 30mK$. To explain the value of the switching current $I_S = 100nA$ compared to the value of the critical current $I_C = 335nA$ we have to take in relation 14 an effective temperature $T_{esc} = 597mK$. With this effective temperature we calculate a reduction factor for the amplitude of the switching current modulation of 0.765 for the 2D dynamics and 0.255 for the 1D dynamics. As noted in the previous section the agreement with the measured amplitude is

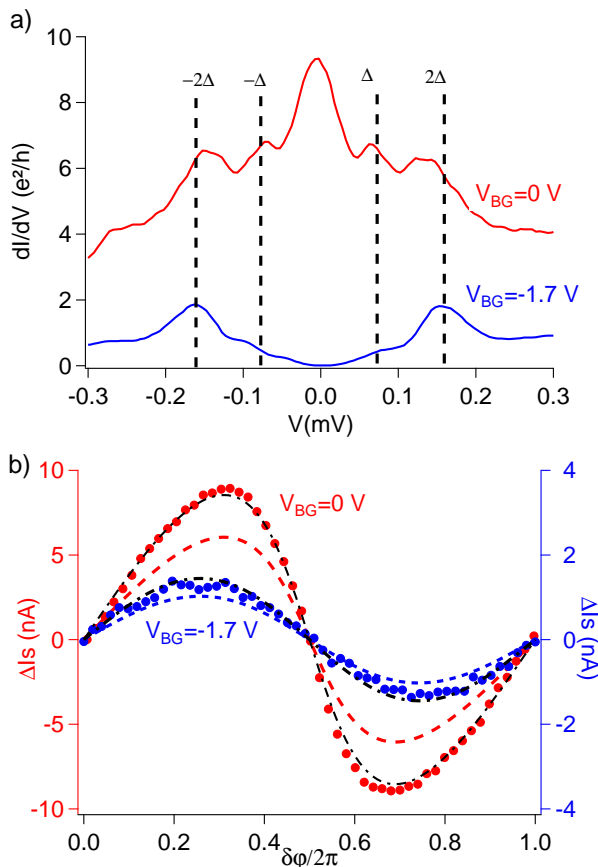


FIG. 11: (a) Differential conductance of the nanotube junction in the superconducting state at two gate voltages, $V_G = 0$ V (in red) corresponding to a high value of normal state conductance and $V_G = -1.7$ V (in blue) corresponding to a low conductance. The two curves show evidence of multiple Andreev reflection. (b) Current phase relation extracted from the experiment at same gate voltages (red and blue circles). The dashed lines are theoretical predictions based on eq. 15 and the 1D dynamics model. The dashed dotted lines correspond to eq.15 scaled by a factor 0.36.

better with the 1D dynamics model and is ≈ 30 % for the two gate values shown. To obtain a better agreement a deeper understanding of the switching in our device is needed. The amplitude of the modulation of the switching current ΔI_S compared to the theoretical value deduced from eq. 15 is found to be 0.36 (black dashed line in fig. 11b). We want to stress that the shape of the expected current-phase relation is well reproduced in the experiment.

V. CONCLUSION

Our detection setup allows to relate the current phase relation measurements to the normal and superconducting states differential conductance dI/dV . This provides a useful way to measure precisely the current phase relation and parameters of more complex system. It might in particular be extremely interesting and challenging to probe the signature of electronic correlation in conjunction with proximity effect or the influence of large spin-orbit interactions.

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